Kharatti Lal Int. Journal of Engineering Research and Applications www.ijera.com
ISSN: 2248-9622, Vol. 5, Issue 12, (Part - 2) December 2015, pp.37-40

RESEARCH ARTICLE OPEN ACCESS

Fuzzy Group Ideals and Rings

Kharatti Lal
Dept. of Applied Science – Mathematics Govt. Millennium polytechnic College Chamba Himachal Pradesh – 176310 (INDIA)

ABSTRACT
This section define a level subring or level ideals obtain a set of necessary and sufficient condition for the equality of two ideals and characterizes field in terms of its fuzzy ideals. It also presents a procedure to construct a fuzzy subrings (fuzzy ideals) from any given ascending chain of subring ideal. We prove that the lattice of fuzzy congruence of group G (respectively ring R) is isomorphic to the lattice of fuzzy normal subgroup of G (respectively fuzzy ideals of R).In Yuan Boond Wu wangrning investigated the relationship between the fuzzy ideals and the fuzzy congruences on a distributive lattice and obtained that the lattice of fuzzy ideals is isomorphic to the lattice of fuzzy congruences on a generalized Boolean algebra. Fuzzy group theory can be used to describe, symmetries and permutation in nature and mathematics. The fuzzy group is one of the oldest branches of abstract algebra. For example group can be used is classify to all of the forms chemical crystal can take. Group can be used to count the number of non-equivalent objects and permutation or symmetries. For example, the number of different is switching functions of n, variable when permutation of the input are allowed. Beside crystallography and combinatory group have application of quantum mechanics.
Key Words: group structure, fuzzy cosset, fuzzy ideal, fuzzy congruence, normal subgroup,

I. INTRODUCTION

One consider the operation of $\cap, \cup, \cdot, \hat{+}, \oplus$. One knows that he group is monoid such that each element possesses one and only one inverse we shall show that the necessary condition for $\left(\underset{\sim}{P}(X)\text{«} ,\right)$ to have a group structure is that M = [0, 1] also have a group structure for an operation corresponding to $\leftrightarrow$. We shall see that in any case M = [0, 1] may be endowed with a group structure for an operation 0 to be defined. Let M = [0, 1] is a vector lattice that may be reduced to a single chain forming a total order. We consider the operation $\wedge(\min)$, $\vee(\max)$, $\cdot(\text{Product})$, $\hat{+}$ (algebric sum), $\oplus$ (disjunctive sum). For each of these operations, one has the associative, property and there exists an identity, which is, depending on the case, 0 or 1; but it is easy to prove that, almost in the same way that for each of these operations, there does not exists an inverse for each element.

II. Fuzzy Characterization of Regularity

At the title suggested this section presents necessary and sufficient for regularity of ring in terms of its fuzzy ideals and fuzzy semiprime ideals.

Recall that a ring R is regular if, for each element x in R, there exists y in R such that xyx = x,

Theorem (1.1) : A ring R is regular fit, every fuzzy ideal of R is idempotent.

Proof: Let Ring R be the regular and let μ be an fuzzy ideal of R. That $R^2 \subseteq \mu$ is obvious. For the reverse inclusion, we first observe that any element x of R can be written as x = xyx, where $y \in R$.

$\Rightarrow$ $\mu^2(x) = \sup(\min(\mu(a), \mu(b))$.

$\Rightarrow$ x = ab

$\Rightarrow$ min (μ(xy)), μ(x), taking a = xy and b = x

$\geq$ Min (H(x), μ(x) = μ(x), since μ is a fuzzy ideal the foregoing arguments yields that $\mu \subseteq \mu^2$;L HENCE $\mu = \mu^2$.

Conversely, let μ and θ be any two fuzzy ideal of R. We argue as follows:

$(\mu \cap \theta)(x) = (\mu \cap \theta)^2(x)$ by the idempotency of $\mu \cap \theta$, $x \in R$

$$= \sup_{x=ab}(\min((\mu \cap \theta)(a),\ (\mu \cap \theta)(b))$$

$$\leq \sup_{x=ab}(\min \mu(a), \theta(a) = (\mu\theta)(x).$$

Hence $\mu \cap \theta \subseteq \mu\theta$ and the equality follows. The regularity of R now at once follows a ring R is regular iff $\sigma\theta = \sigma \cap \theta$, where σ and θ are any two fuzzy ideal of R.

Theorem (1.2): A ring R is regular iff, every fuzzy ideal of R is fuzzy semiprime.

Proof: First, we assume that R is a regular and take any fuzzy ideal μ of R. Let σ be any fuzzy ideal of R such that $\sigma^n \subseteq \mu$ where $n \in Z_+$. The maximum theorem implies that $\sigma_n = \sigma$, and so $\sigma \subseteq \mu$. Hence μ is a fuzzy semiprime. As regards the converse, we assume that every fuzzy ideal of R is fuzzy semiprime from lemma it follows that. Let A be any nonempty proper subset of a ring R and let $x \in R$. Then the following statements are true (i) $I_m((¥_A)^m) = \{0, 1\}$) and (ii) $(¥_{\langle x\rangle})^m = ¥_{\langle x\rangle}m$ where $m \in Z_+$ since $(¥_{\langle x\rangle})^2 = ¥_{\langle x^2\rangle}$ for any x in R. So that $¥_{\langle x\rangle} \subseteq ¥_{\langle x^2\rangle}$. Therefore $¥_{\langle x^2\rangle}$; and hence $x \in \langle x^2\rangle$

where $x = rx^2 = xrx$ for some $r \in R$

Thus, R is regular ring

Remarks (2.1): If μ is any fuzzy ideal of a Boolean ring, then $\sqrt{\mu} = \mu$, since every fuzzy ideal of a such ring is fuzzy semiprime. Consequently in any Boolean ring-fuzzy maximal ideal, non-constant fuzzy prime ideals, non constant fuzzy primarily ideals, non-constant fuzzy semiprimary ideals – coincide. That is if μ is any non-constant fuzzy ideals of a Boolean ring R, then we have μ is fuzzy maximal $\Leftrightarrow$ μ is fuzzy prime $\Leftrightarrow$ μ is fuzzy primary $\Leftrightarrow$ μ is fuzzy semiprimary.

It is worth noting that $¥_{\langle x\rangle} \subseteq_{\langle x\rangle}$. Therefore $¥_{\langle x\rangle} = ¥_{\langle x^2\rangle}$: hence $x \in \langle x^2\rangle$ so that $x = rx^2 = xrx$, for some $r \in R$ thus R is regular permits us to exclude fuzzy semiprime ideal from the above chain of equivalence because otherwise every non-constant of fuzzy ideals of a Boolean ring will be fuzzy maximal. This in turn would imply every non-zero Boolean in is a field which is blatantly We show this for $\wedge$. Consider a pair $(a, b) \in M \times M$

$M = [0, 1]$ and such that $0 < a < b < 1$. The identity of $\wedge$ is 1. Does there exists an a or ab such that

$A \wedge B = 1$,

This is impossible:

$A \wedge b = a < 1$

on the other hand, if one taken $M = \{0, 1\}$. One finds that a group is possible one finds that a group is possible

(i)

$\wedge$	0	1
0	0	0
1	0	1

Table (i)

(ii)

$\vee$	0	1
0	0	1
1	1	1

Table (ii)

(i) This is not a group

The identity is 1, but:

$0 \wedge 0 = 0$,

$0 \wedge 1 = 0$,

$1 \wedge 0 = 0$,

$1 \wedge 1 = 1$,

0 does not have an inverse

(ii) This is not a group

The identity is 0, but:

$0 \vee 0 = 0$,

$0 \vee 1 = 1$,

$1 \vee 0 = 1$,

$1 \vee 1 = 1$,

1 does not have an inverse

(iii)

	0	1
0	0	1
1	1	0

Table (iii)

This is a group

The identity is 0

The inverse of 0 is 0

The inverse is 1 is 1

(iv)

	0	1
0	1	0
1	0	1

Table (iv)

This is a group

The identity is one (1)

The inverse of 0 is 0

The inverse of 1 is 1

From table does not obtain a group $\wedge$ or $\vee$. On the contrary one does obtain a group if one takes the operation $\oplus$.One also obtain group if one considered the operation$\oplus$. One also obtain group if one considered the operation $\oplus$ inverse disjunctive sum. We note that the two groups $\oplus$ and $\overline{\oplus}$ are isomorphic + by permitting 0 and 1; a single group may represent the two. It follows from this that if one considers any one of the operation $\cap$, $\cup$, ., $\hat{+}$, $\oplus$, and $M = [0, 1]$ one may not give $\underset{\sim}{P}(E)$,« a group structure. It take $M = [0, 1]$, it is only with $\oplus$ or what amount of the same things with $\overline{\oplus}$ that one may from a group. We consider as an example the ordinary group formed

$E = \{x_1, x_2, x_3\}$

$\oplus$

	000	001	010	011	100	101	110	111
000	000	001	010	011	100	101	110	111
001	001	000	011	010	101	100	111	110
010	010	011	000	001	110	111	100	101
011	011	010	001	000	111	110	101	100
100	100	101	110	111	000	001	010	011
101	101	100	111	110	001	000	011	010
110	110	111	100	010	010	011	000	001
111	111	110	101	010	011	010	001	000

Table (V)

If one puts abc = $\{(x_1 \mid a), (x_2 \mid b), (x_3 \mid c)\}$
In order to simplify writing with
$a, b, c \in \{0, 1\}$,
One obtain the group represented in table (v), the identity is 000 and each element abc has itself for an inverse. This group $\left(\underset{\sim}{P}(E), \text{«}\right)$ has been represented in the figure:

	0	1	2	3	4	5	6	7
0	0	1	2	3	4	5	6	7
1	1	0	3	2	5	7	7	6
2	2	3	0	1	6	7	4	5
3	3	2	1	0	7	6	5	4
4	4	5	6	7	0	1	2	3
5	5	4	7	6	1	0	3	2
6	6	7	4	5	2	3	0	1
7	7	6	5	4	3	2	1	0

Table (v i)

By replacing the binary number abc by their corresponding decimals. One correctly notices. Certain properties (Subgroup, Latin square etc.) these properties are very general for these groups constructed with $\oplus$ i.e. all the corresponds structures configurations of a membership set M, which we shall generalize by examining other totally ordered configuration for him.

III. FUZZY CONGRUENCE ON GROUPS AND RINGS

Definition: (2.1) A fuzzy equivalence relation q on a group G is a fuzzy congruences on G if the following conditions are satisfied for all x, y z + in G (G1): $\theta(x \leftrightarrow z, y \leftrightarrow t) \geq \theta(x, y) \wedge \theta(z \leftrightarrow x)$

(b) A fuzzy equivalence relation θ on a ring is a fuzzy ideal on R, if the following conditions are satisfied for all x, y, z, t in R (R_1) : $\theta(x + z; y + t) \geq \theta(x, y) \wedge \theta(z, t)$;

Lemma: If $\theta \in F \subset (G)$ and $x, y\ z \in G$ then
(i) $\theta(x, y) \geq \theta(x \leftrightarrow y, y \leftrightarrow z) \wedge \theta(z \leftrightarrow x, x \leftrightarrow y)$
(ii) $\theta(x^{-1}, y^{-1}) = \theta(x, y)$

Proof: $\theta(x, y) = \theta(x \leftrightarrow z \leftrightarrow z^{-1}, y \leftrightarrow z \leftrightarrow z^{-1}) \geq (x \leftrightarrow y, y \leftrightarrow z) \wedge \theta(z^{-1}, z^{-1})$
i.e. $= \theta(x \leftrightarrow z, y \leftrightarrow z)$
i.e. $= \theta(x, y) = \theta(z^{-1} \leftrightarrow z \leftrightarrow x; z^{-1} \leftrightarrow z \leftrightarrow y)$
$= \theta(z^{-1}, z^{-1}) \wedge \theta(z \leftrightarrow x), (z^{*} \leftrightarrow y)$
$= \theta(x, y) \geq \theta(x \leftrightarrow y, y \leftrightarrow z) \cap \theta(z \leftrightarrow x, z \leftrightarrow y)$

Point II
$\Rightarrow \quad \theta(x^{-1}, y^{-1}) \geq \theta(x \leftrightarrow x^{-1}, x \leftrightarrow y^{-1})$
$\geq \theta(e, x \leftrightarrow y^{-1}) \geq \theta(y, x) = (x, y)$
On the other hand $\theta(x^{-1}y^{-1}) \geq \theta(x, y)$
Implies that $\theta(x,y) \geq \theta(x^{-1}, y^{-1})$. Since x and y are arbitrary element $\Rightarrow \theta(x^{-1}, y^{-1}) = \theta(x, y)$.

(2.2) Fuzzy Ideal Generated by a Fuzzy Subset
Let μ be any fuzzy subset of a ring R the smallest fuzzy ideals of R containing μ is called the fuzzy ideals generated by μ in R Z_x and is denoted by $< \mu >$

Proposition: Let A be any fuzzy subset of a ring R, the $\langle ¥_A \rangle = ¥_{\langle A \rangle}$

Theorem (2.3): Let μ be any fuzzy subset of a ring R. Then the fuzzy subset μ^* of R defined by,
$\mu^*(x) = \sup \{k \mid x \in \langle \mu_k \rangle \}$.

In the fuzzy subring (fuzzy ideals) generated in the μ in R is the ideal generated by μ_k in other words $\mu^*(x) = t$ whenever $x \in \langle \mu_t \rangle \notin$ and $x \notin \mu_s \leftrightarrow$ for all $\delta > t$

Proposition: The set of all fuzzy idealsμ's of a ring R is a complete lattice under the relation ⊆.The sup and if of any subfamily.
$\{\mu_i \mid i \in \Omega\}$ of fuzzy ideals are
$< \cup \{\mu_I \mid I \in \Omega\} > \cap \{4\mu_I \mid I \in \Omega\}$ respectively.

IV. Conclusion

We observe that the notion of fuzzy ideal of ring, fuzzy sub–space of a vector space, fuzzy normal subgroup of a group, fuzzy subuniverse of a universal algebra and fuzzy equivalence fit very comfortably in the frame work of our general theory of algebraic fuzzy system. For any set S of sub set of X, we written S_ϕ to denote the set S U{ ϕ}.note that S is closure set system on X. We discuss a modularity of a lattice S and FS. Also we give a method to construct a fuzzy S-subset of X, satisfying the certain condition s we apply the general theory of fuzzy S system to certain cases ,Where s stands for the set of all ideals of a ring normal subgroup of a group , subspace of a vector space ,sub universal of a universal algebra equivalence relation on a universal algebra throughout this paper 1 = (1,∧ , ∨, 0,1) stands for a non – trivial complete, Brouwerian lattice i.e a complete lattice satisfying the infinite meet distributive law $\alpha \wedge (V_{\beta \varepsilon M} B) = (V_{\beta \varepsilon M} (\alpha \wedge \beta)$ for all M is the set of L and $\alpha \varepsilon L$) an L – fuzzy subset of X is a mapping of X into L, if L is a unit interval [0, 1] of real number s, there are the usual fuzzy subset of X originally introduced by Zadeh in pioneering work [1 6].

REFERENCE

[1.] Rosenfeld SA, Fuzzy group J Math Anal. Appl. 35: 512-517, 1971.
[2.] Bourbaki, Elements of mathematics' commutative algebra, Springer Verlag, New York, 1989.
[3.] Gerla, G, ‘‘Code theory and fuzzy subrings-semigroup. J. Math. Anal ,'' and Appl. 128: 362-369, 1987.
[4.] Kandel A, "Fuzzy Mathematical technology" with application. Addison-Wasely Pub. Co. 1986.
[5.] Mordeson J.N. ‘‘Fuzzy coefficient field of fuzzy subrings fuzzy set and system''.58: 227-337, 1993.
[6.] Mordeson J. and Alkhamess Y. Fuzzy localized subrings inform Sci. 99: 183-193, 1997.
[7.] Gerla, G, ‘‘Code theory and fuzzy subrings – semigroup''. J. Math. Anal, and Appl. 128: 362-369, 1987.
[8.] Godon J.S. ‘‘The theory of semirings with application in mathematics and theoretical computer science'', Pitman Monographs and Surveys in pure and applied mathematics. 54, Longman Scientific and Technical John Wiley and Sons Inc. New York, 1992.